Wollo University
Kombolcha Institute of Technology
School of Mechanical & Chemical Engineering

# Chapter 4: Flow Measurement

Prepared by: Wubishet Degife.
E-mail: wubesvictory@gmail.com

# Outline lecture notes

- Pressure measurement
- Velocity measurement
- Discharge measurement
- Viscosity measurement
- Density measurement

# Introduction

- Why measurement is important???

Engineering is a creative and learning profession. If engineers are to create, they must experiment and open the new frontiers of information through well-planned and suitably conducted experiments. There would thus be a need to measure the physical entities such as displacement, velocity, pressure, force, elapsed time etc. in the operating devices and machines. Experimentation is considered to be the cornerstone in the field of engineering design, research and development projects. In industry too, there is need for the measurement and control of the physical conditions required for mass production and high quality products. Similarly in commercial organisations, the measurement of water and electricity supplied to a consumer is a must.

The instruments for measurement, control and transmission find such a wide and varied use that they have become an essential feature of technological operations and modern day-to-day life. It would be difficult to think of any man made article whose manufacture did not at some stage involve measurement. There are instruments to control the flight of man-made satellites, to probe the mysteries of outer space and to transit the relative information. Nearer at home, we use instruments to control the temperature of our homes, and to preserve food in refrigerators and cold storages. Our automobiles are equipped with instruments to measure speed, condition of battery and the amount of gasoline in fuel tank.

In this chapter an attempt has been made to present the principles and phenomenon of fluid mechanics which are adopted for the measurements of **fluid pressure**, **flow velocity**, **flow rate** or **discharge rate**, **viscosity of fluids** and **density of fluids** using different measuring instruments/devices and their working principles

# Pressure measurement

- The pressure of a fluid is measured by the following devices
  1. Manometers and
  2. Mechanical gauges

**Manometers**: are devices which are used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

- They are classified as:
  A. Simple manometers and
  B. Differential manometers

M**echanical gauges**: are devices used for measuring the pressure by balancing the fluid column by the **spring** or **dead weight**.

- The commonly used mechanical gauges are:

(*a*) Diaphragm pressure gauge,
(*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and
(*d*) Bellows pressure gauge.

**The simple manometer**: consists of a glass tube having one of its end connected to a point where pressure is to be measured and the other end remains open to the atmosphere.

- The common types of the simple manometers are:

1. Piezometer,`
2. U-tube Manometer, and
3. Single Column Manometer.

**Piezometer**:

- It is the simplest form of manometer used for measuring gauge pressures.
- One end of this manometer is connected to the point where pressure is to be measured and the other end is open to the atmosphere as shown in fig 4.1.
- The rise of liquid gives the **pressure head** at that point.
  - ✓ For example, if u want to measure the pressure at point A given the fluid is water, the height of the liquid water rises h meter in piezometer tube, then the pressure at A would be:

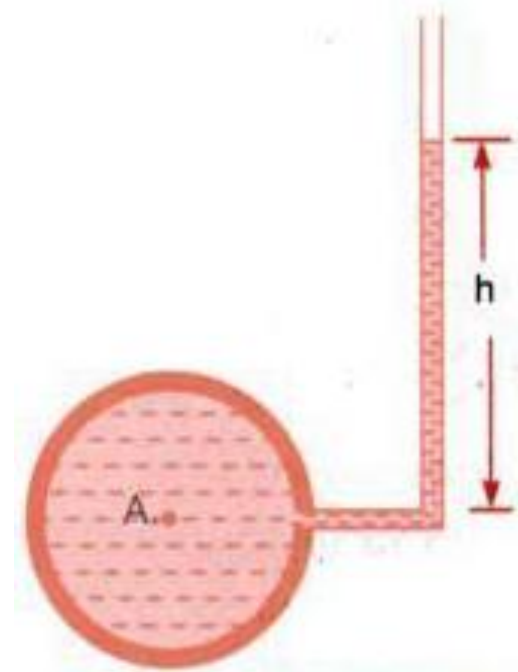
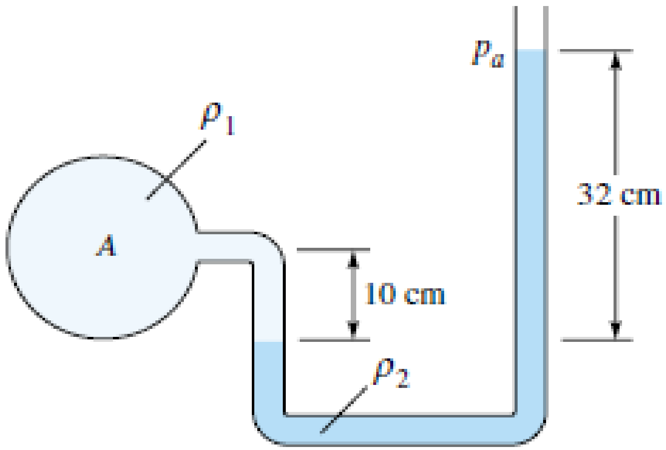


Fig. 4.1: Piezometer

$$P_A = \rho g h + P_{atm} = \rho g h \quad \textit{since it is calibrated to read zero atmospheric presure}$$

**Example 1**. The piezometer shown in fig. fluid 1 is oil (SG = 0.87) and fluid 2 is glycerin at 20°C. If $P_a$ = 98 kPa, determine the absolute pressure at point A, given specific weight of water is 9790N/m$^3$ and specific weight of glycerin at 20°C is 12360N/m$^3$.



**Solution for example 1**

$P_C = P_D$, but

$P_C = P_A + \rho_1 * g * 0.1m$, but $\rho_1 * g = S.G_{oil} *$Specific weight of water = $0.87 * 9790 N/m^3 = 8517.3 N/m^3$

$P_C = P_A + 8517.3 * 0.1m$

$P_C = P_A + 851.73pa$

$P_D = P_{atm} + \rho_2 * g * 0.32m$

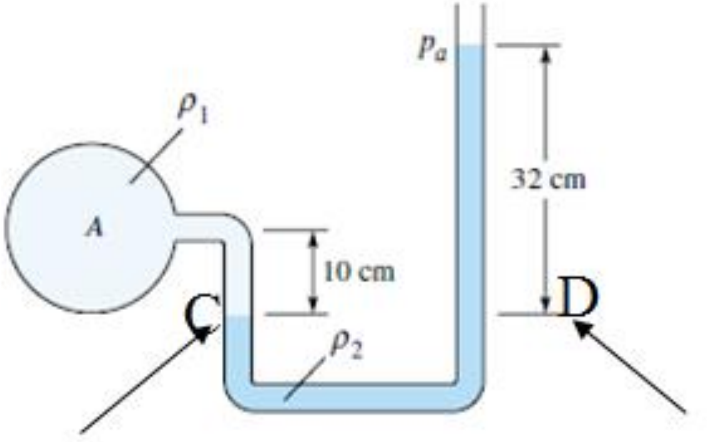
$P_D = 98kpa + \rho_2 * g * 0.32m$, but $\rho_2 * g = 12360 N/m^3$

$P_D = 98000pa + 12360 N/m^3 * 0.32m = 101955.2 N/m^2$

Thus, $P_C = P_D$

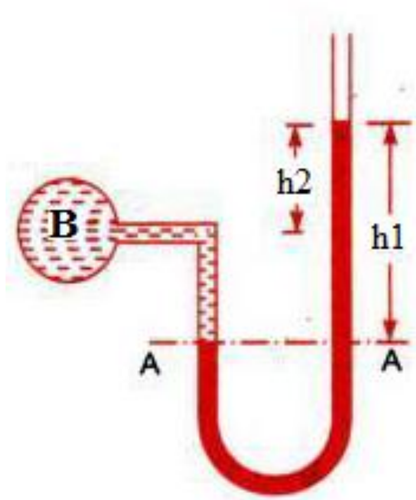
$P_A + 851.73pa = 101955.2 N/m^2$

$\mathbf{P_A = 101103.47pa}$

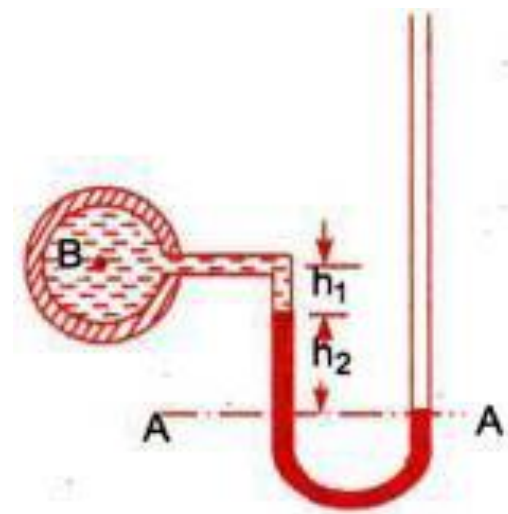


**U-tube manometer**: it consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and the other end remains open to the atmosphere as shown in fig. 4.2.

- The tube generally contains **mercury or any other liquid** whose specific gravity is **greater than** the specific gravity of the liquid whose pressure is to be measured.




(a) For gauge pressure




(a) For vacuum pressure

Fig. 4.2: U-tube manometer

**(a) For gauge pressure**: Let B be the point at which pressure is to be measure and let whose value is P. Then, at the datum line A-A, the left and right side of the U-tube has the same pressure because it is at the same depth.

*Assuming the liquid in the left column has density* $\rho_1$

*Thus pressure above A – A in the left column* $= P + \rho_1 g(h_1 - h_2)$

*and the pressure above A – A in the right column* $= \rho_2 g h_1 + P_{atm}$

$= \rho_2 g h_1$ *if atmospheric pressure is ignored*

Hence these two pressures (right and left column of the U-tube at point A-A) are equal, then equating them would give:

$$P + \rho_1 g(h_1 - h_2) = \rho_2 g h_1$$

$$\textit{Thus} \qquad P = \rho_2 g h_1 - \rho_1 g(h_1 - h_2)$$

(b**) For vacuum pressure**: for measuring vacuum pressure, the level of the heavy liquid in the manometer will be shown in fig 4.2b, then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$ ← If $P_{atm}$ is ignored

$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1).$



**<u>Note that</u>**: Most pressure measurement devices are calibrated to read zero atmospheric pressure.

# Cont…

**Single column manometer**:

- A single column manometer is a modified form of a U-tube manometer in which a reservior a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in fig 4.3.
- Due to large cross-sectional area of the reservior, for any variation in pressure, the change in the liquid level in the reservior will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined.
- Thus, there are two types of single column manometer as:
  - ✓ Vertical single column manometer, and
  - ✓ Inclined single column manometer

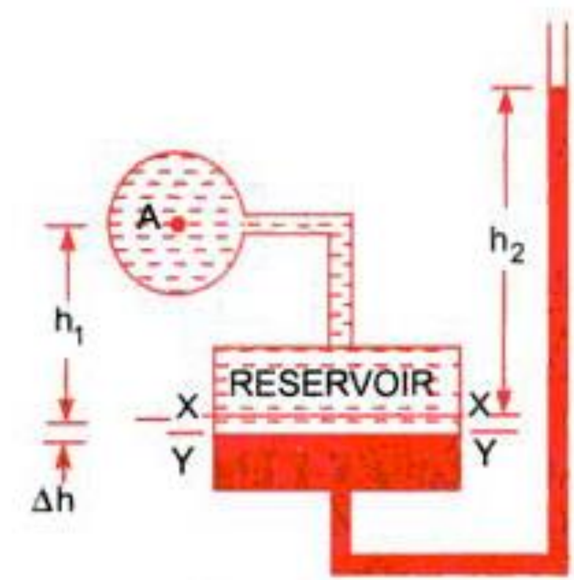



Fig. 4.3: vertical single column manometer

Vertical single column manometer:

Fig. **4.3** shows the vertical single column manometer. Let *X-X* be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at *A*, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let $\Delta h$ = Fall of heavy liquid in reservoir

$h_2$ = Rise of heavy liquid in right limb

$h_1$ = Height of centre of pipe above *X-X*

$p_A$ = Pressure at *A*, which is to be measured

$A$ = Cross-sectional area of the reservoir

$a$ = Cross-sectional area of the right limb

$S_1$ = Sp. gr. of liquid in pipe

$S_2$ = Sp. gr. of heavy liquid in reservoir and right limb

$\rho_1$ = Density of liquid in pipe

$\rho_2$ = Density of liquid in reservoir

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \qquad A \times \Delta h = a \times h_2$$

$$\therefore \qquad \Delta h = \frac{a \times h_2}{A} \qquad \text{.................................. (i)}$$

Now consider the datum line $Y$-$Y$ as shown in Fig.

Then pressure in the right limb above $Y$-$Y$ $= \rho_2 \times g \times (\Delta h + h_2)$

Pressure in the left limb above $Y$-$Y$ $= \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

or
$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$
$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation (*i*), $\quad \Delta h = \dfrac{a \times h_2}{A}$

$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

As the area $A$ is very large as compared to $a$, hence ratio $\dfrac{a}{A}$ becomes very small and can be neglected.

Then $p_A = h_2\rho_2 g - h_1\rho_1 g$ ............................... (ii)

From equation (ii), it is clear that as $h_1$ is known and hence by knowing $h_2$ or rise of heavy liquid in the right limb, the pressure at A can be calculated.

**Inclined single column manometer:**

Fig. **4.4** shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

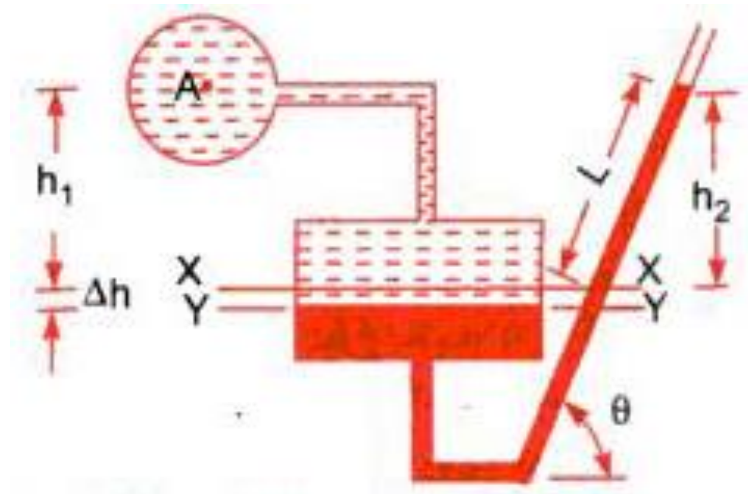



Fig. 4.4: inclined single column

Let $L$ = Length of heavy liquid moved in right limb from $X$-$X$

$\theta$ = Inclination of right limb with horizontal

$h_2$ = Vertical rise of heavy liquid in right limb from $X$-$X = L \times \sin\theta$

From equation ( ii ), the pressure at $A$ is

$$p_A = h_2\rho_2 g - h_1\rho_1 g.$$

Substituting the value of $h_2$, we get

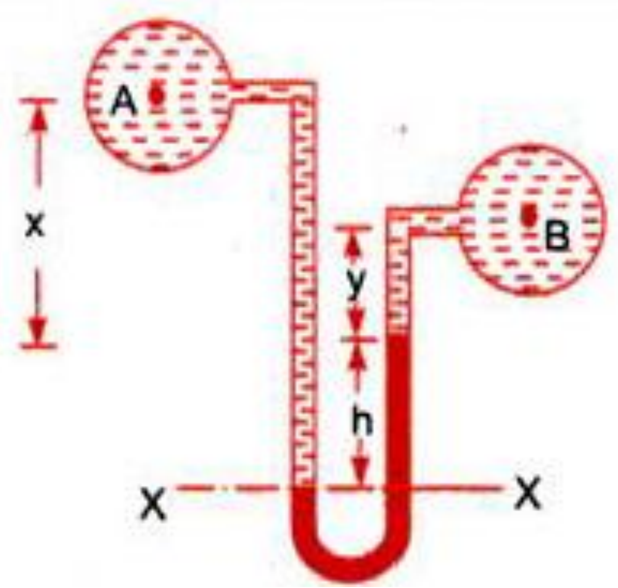
$$P_A = L\sin\theta\rho_2 g - h_1\rho_1 g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

**Differential manometers:**

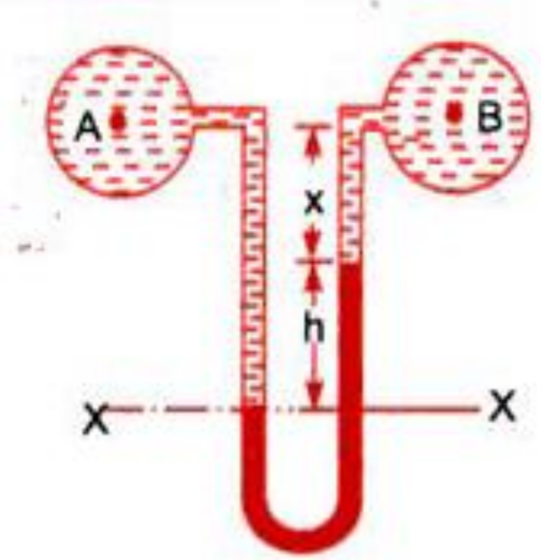
Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

**U-tube differential manometer**: fig 4.5 shows the differential manometer of U-tube type




(a) Two pipes A and B at different levels




(b) Two pipes A and B at same levels

Fig 4.5: U-tube differential manometers

**Fig** 4.5 (*a*). Let the two points $A$ and $B$ are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at $A$ and $B$ are $p_A$ and $p_B$.

Let $h$ = Difference of mercury level in the U-tube.

$y$ = Distance of the centre of $B$, from the mercury level in the right limb.

$x$ = Distance of the centre of $A$, from the mercury level in the right limb.

$\rho_1$ = Density of liquid at $A$.

$\rho_2$ = Density of liquid at $B$.

$\rho_g$ = Density of heavy liquid or mercury.

Taking datum line at $X$-$X$.

Pressure above $X$-$X$ in the left limb $= \rho_1 g(h + x) + p_A$

where $p_A$ = pressure at $A$.

Pressure above $X$-$X$ in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where $p_B$ = Pressure at $B$.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx \quad \text{.................................} \text{(iv)}$$

$\therefore$ Difference of pressure at $A$ and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

**Fig.** 4.5 (*b*). *A* and *B* are at the same level and contains the same liquid of density $\rho_1$. Then

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g (h + x)$$

$$= g \times h(\rho_g - \rho_1). \quad \text{.............................................} \text{(v)}$$

**Inverted U-tube differential manometer:**

- It consists of an inverted U-tube containing a light liquid.
- The two ends of the tube are connected to the points whose difference of pressure is to be measured.
- Fig 4.6 shows an inverted U-tube differential manometer connected to the two points A and B. As can be seen in the figure, the pressure at A is more than (greater than) the pressure at B since point A is deeper than point B

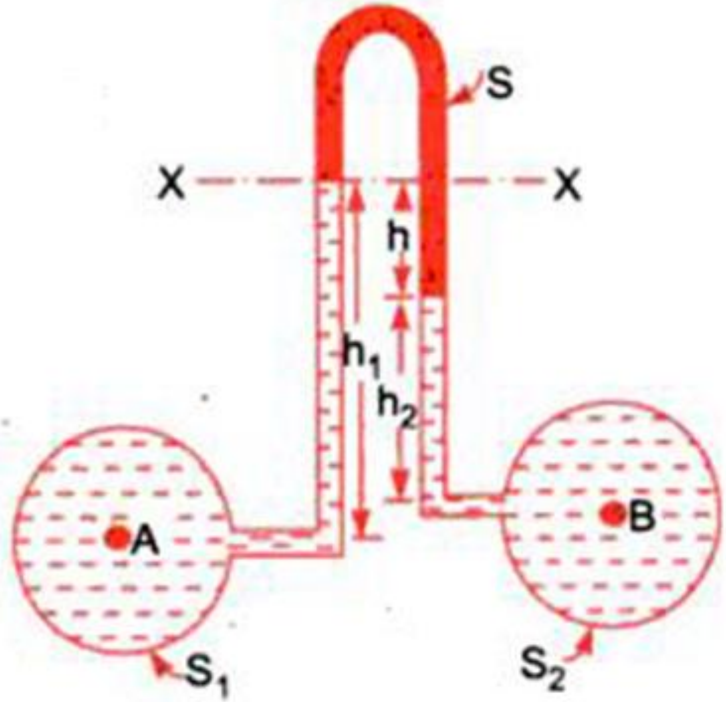



Fig. 4.6: Inverted U-tube differential manometer

Let $h_1$ = Height of liquid in left limb below the datum line $X$-$X$

$h_2$ = Height of liquid in right limb

$h$ = Difference of light liquid

$\rho_1$ = Density of liquid at $A$

$\rho_2$ = Density of liquid at $B$

$\rho_s$ = Density of light liquid

$p_A$ = Pressure at $A$

$p_B$ = Pressure at $B$.

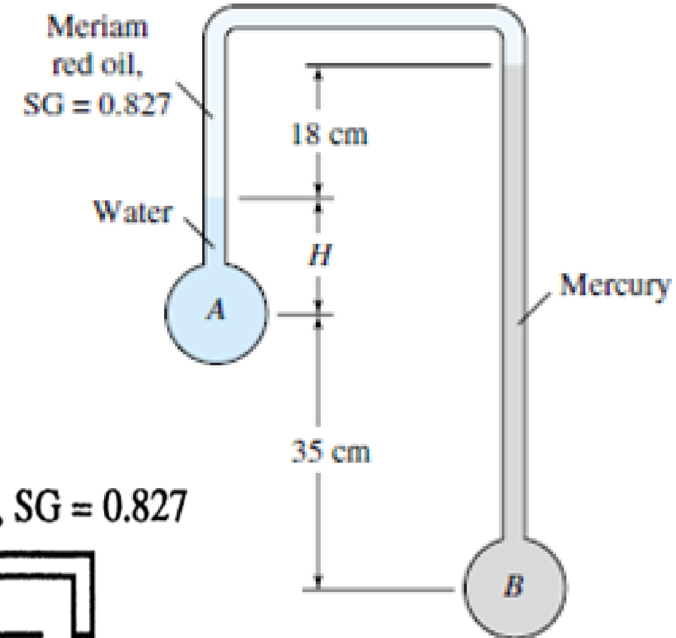
Taking $X$-$X$ as datum line. Then pressure in the left limb below $X$-$X$ = $p_A - \rho_1 \times g \times h_1$

Pressure in the right limb below $X$-$X$ = $p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad \text{......(vi)}$$




**Example 2**: For the inverted manometer shown in fig., all fluids are at 20°C. If $P_B$ - $P_A$ = 97 kPa, what must the height H be in cm? given specific weight of mercury is 133100N/m$^3$

**Solution**

For the manometer of Fig. P2.32, all fluids are at 20°C. If $p_B - p_A = 97$ kPa, determine the height H in centimeters.

**Solution:** Gamma = 9790 N/m$^3$ for water and 133100 N/m$^3$ for mercury and (0.827)(9790) = 8096 N/m$^3$ for Meriam red oil. Work your way around from point A to point B:

$$p_A - (9790\ \text{N/m}^3)(\text{H meters}) - 8096(0.18)$$

$$+133100(0.18 + H + 0.35) = p_B = p_A + 97000.$$

$$\text{Solve for} \quad H \approx 0.226\ \text{m} = \mathbf{22.6\ cm} \quad \textit{Ans.}$$

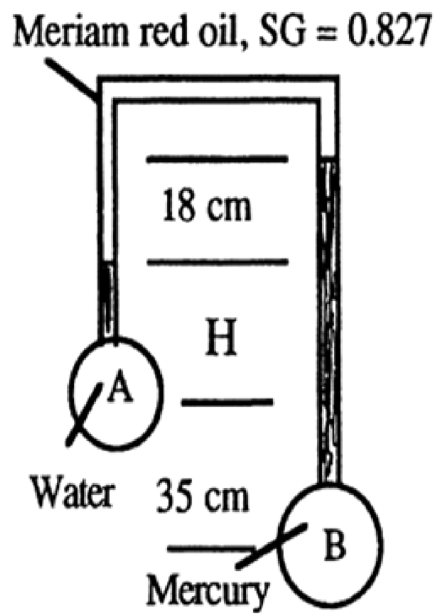



Fig. P2.32

# Velocity measurement

- Velocity constitutes an important parameter in **kinematics** and **dynamics** of fluid flow. Thus, its measurement is quite important.
- Velocity measuring devices may be classified as:
  - ✓ Three-cup anemometer;
  - ✓ Savonius rotor;
  - ✓ Tturbine mounted in a duct;
  - ✓ Free-propeller meter;
  - ✓ Hot-wire anemometer; (f) hot-film anemometer;
  - ✓ Pitot-static tube;
  - ✓ Laser - Doppler Anemometer, each is shown in figure 4.7 below
- Thus, basically we can classify velocity measuring devices as:
  - ✓ Instruments like the **pitot - tube** and **hot wire anemometers** which measure the local velocity at a point in the channel or duct through which the fluid is flowing.
  - ✓ Instruments like the **cup** and **vane anemometers** which measures the average velocity of fluid flow.

# Cont…

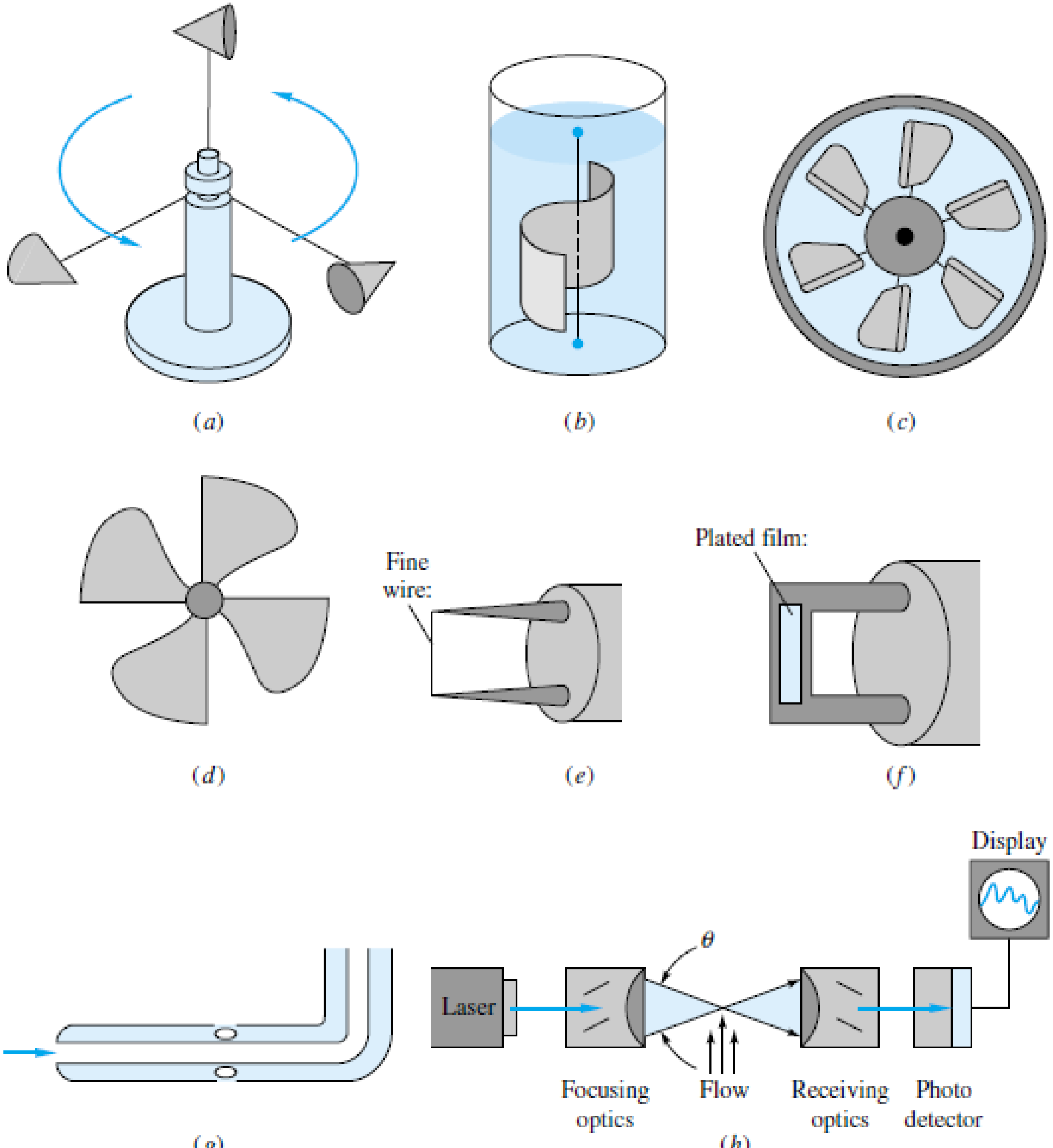



**Fig. 4.7: Eight common velocity** meters: (a) three-cup anemometer; (b) Savonius rotor; (c) turbine mounted in a duct; (d) free-propeller meter; (e) hot-wire anemometer; (f) hot-film anemometer; (g) pitot-static tube; (h) laser doppler anemometer.

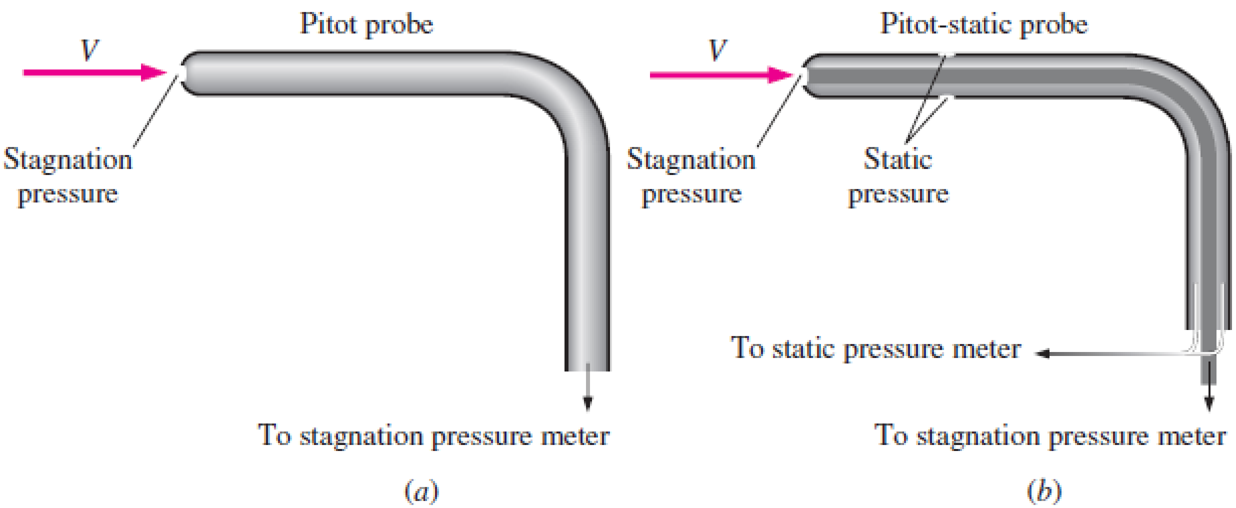
# Cont…

A very important class of velocity measuring devices consists of constriction meters. These involve venturimeters, orifice plates, nozzles and weirs ; all of which measure the average velocity of flow from which the volumetric flow rate can be computed.

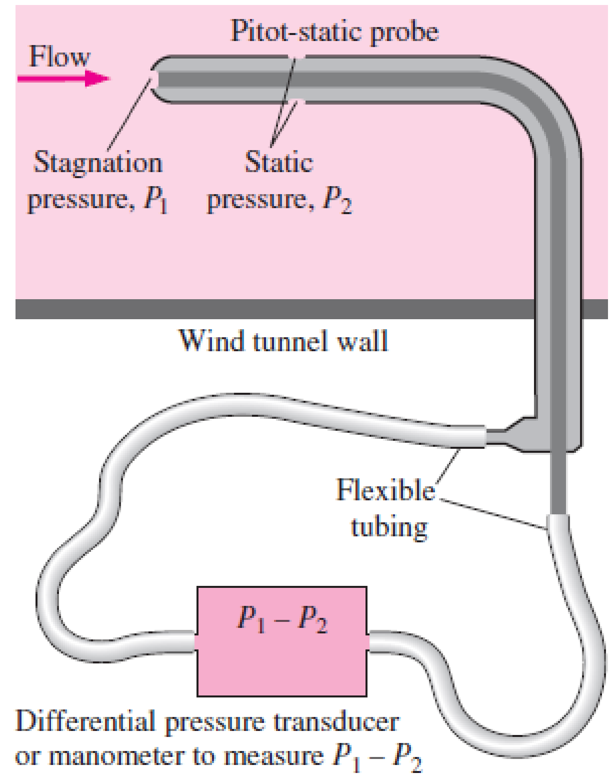
1. **The Pitot static tube**:

- Pitot probes (also called Pitot tubes) and Pitot-static probes, named after the French engineer Henri de Pitot (1695–1771), are widely used for flow velocity measurement.
- Is a slender tube aligned with the flow (Figs. 4.7g) can measure local velocity by means of a pressure difference.
- It has **sidewall holes** to measure the **static pressure** $P_s$ in the moving stream and a hole in the front to measure the **stagnation pressure** $P_o$, where the stream is decelerated to zero velocity.
- Instead of measuring $P_o$ or $P_s$ separately, it is customary to measure their difference with a **transducer**, as in Fig. 4.8.
- A Pitot probe is just a tube with a pressure tap at the stagnation point that measures stagnation pressure, while a Pitot-static probe has both a stagnation pressure tap and several circumferential static pressure taps and it measures both stagnation and static pressures (Figs. 4.8a, b and 4.8c).

# Cont…




(a) A Pitot probe measures stagnation pressure at the nose of the probe, while (b) a Pitot-static probe measures both stagnation pressure and static pressure, from which the flow speed can be calculated.




**(c)** Measuring flow velocity with a Pitotstatic probe. (A manometer may also be used in place of the differential pressure transducer.)

**Fig. 4.8: Pitot-static tube for combined** measurement of static and stagnation pressure in a moving stream.

**Note that**: The flow around the probe is nearly **frictionless** and **Bernoulli's relation**, applies with good accuracy.

# Cont…

- It consists of a **slender double-tube** aligned with the flow and connected to a **differential pressure meter**. The inner tube is fully open to flow at the nose, and thus it measures the stagnation pressure at that location (point 1). The outer tube is sealed at the nose, but it has holes on the side of the outer wall (point 2) and thus it measures the static pressure.

- In its elementary form, a **pitot tube** consists of an **L-shape tube**; a tube bent through 90° and with ends **unsealed**.
  - ✓ One limb called the **body** is inserted into the flow stream and **aligned with the direction of flow** whilst the other limb called the **stem** is **vertical and open to the atmosphere** as shown in fig. 4.9.

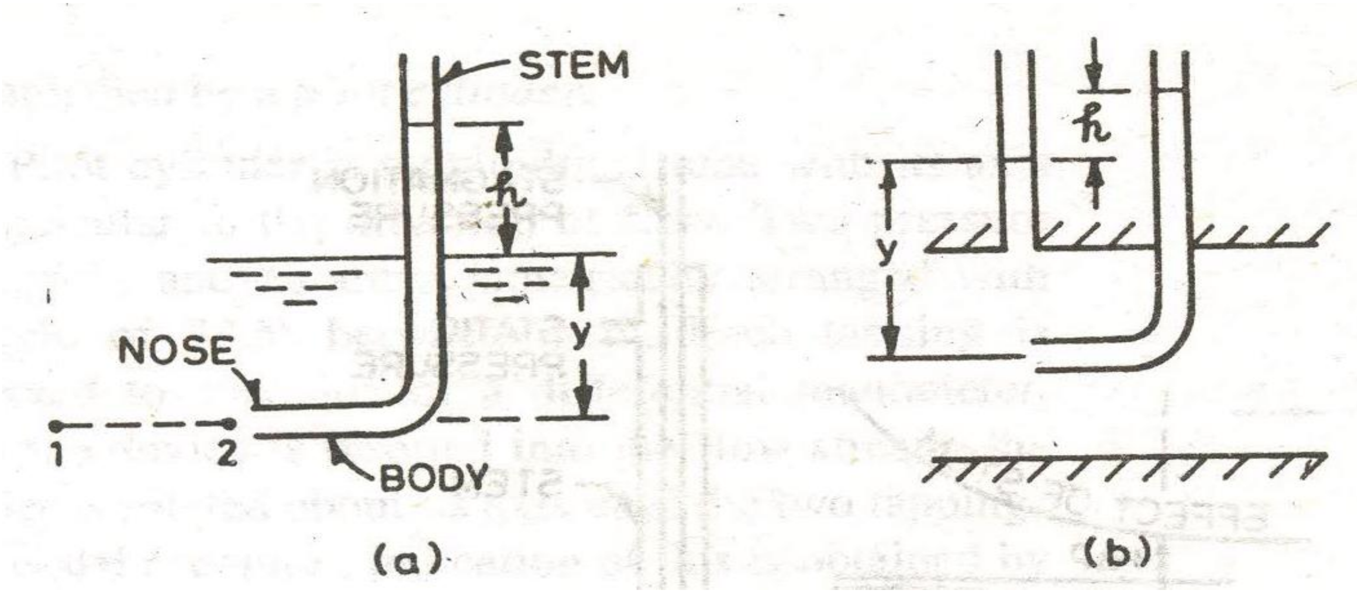



Fig. 4.9: Pitot static probes

- Applying the **Bernoulli's equation** to **point 1** (a point upstream from the submerged end of the tube) and **point 2** ( a point at the tip or nose of the tube itself)

Cont…

$$\frac{V_1{}^2}{2g}+\frac{P_1}{\rho g}+y_1=\frac{V_2{}^2}{2g}+\frac{P_2}{\rho g}+y_2 \quad \dots\dots\dots\dots\dots(a)$$

$Where\ P_1\ is\ the\ static\ pressure, i.e.\ P1\ = P_s\ ,$

$and\ P_2\ is\ the$ $\boldsymbol{stagnation}$ $or$ $\boldsymbol{total\ pressure}$ where the fluid is brought to rest

$i.e.\ \ P_2 = \ P_o\ \ or\ in\ some\ books\ it\ is\ designated\ as\ P_t$

- Now at **the stagnation point** (i.e. point 2 or at the nose or tip of the tube) the flowing fluid is brought to **rest** i.e $V_2 = 0$. Further points 1 and 2 lie at the same elevation datum i.e. $y_1 = y_2$, Equation (a) further simplified as:

$$\frac{P_2}{\rho g}-\frac{P_1}{\rho g}=\frac{V_1{}^2}{2g}$$

Re-arranging and simplifying we get

$$V_1 = \left(2\left(\frac{P_2-P_1}{\rho}\right)\right)^{\frac{1}{2}} \quad OR \quad V_1 = \left(2\left(\frac{P_o-P_s}{\rho}\right)\right)^{\frac{1}{2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(b)$$

Where $V_1$ is the free stream fluid velocity.

This is the **Pitot formula**, named after the **French engineer who designed the device** in 1732.

From fig. 4.9b;

$$\textit{The static pressure } P_s = \rho g y \quad \textit{and the stagnation pressure } P_o = \rho g(y+h)$$

Where h is the rise of liquid level in the stem above the free surface

Thus up on substitution the value of $P_s$ and $P_o$ in Equation (b), we obtain:

$$V_1 = \left(2\frac{\rho g(y+h) - \rho g y}{\rho}\right)^{\frac{1}{2}} = (2gh)^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

- Evidently, **equation (b) and (c)** are equally valid for determining the flow velocity at a point in the pipe line

**2. Hot wire anemometer**:

- A very **fine wire** (d = 0.01 mm or less) heated between two small probes, as in Fig. 4.7e, is ideally suited to measure **rapidly fluctuating flows** such as the turbulent boundary layer.
- The idea dates back to work by L. V. King in 1914 on heat loss from long thin cylinders.
- If electric power is supplied to heat the cylinder, the loss varies with flow velocity across the cylinder according to King's law

$$q = I^2 R \approx a + b(\rho V)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

# Cont…

**where**

**q** – is the power output I – is the current V – is the flow velocity, and

R – is the resistance

**n ≈ 1/3** at very **low Reynolds numbers** and

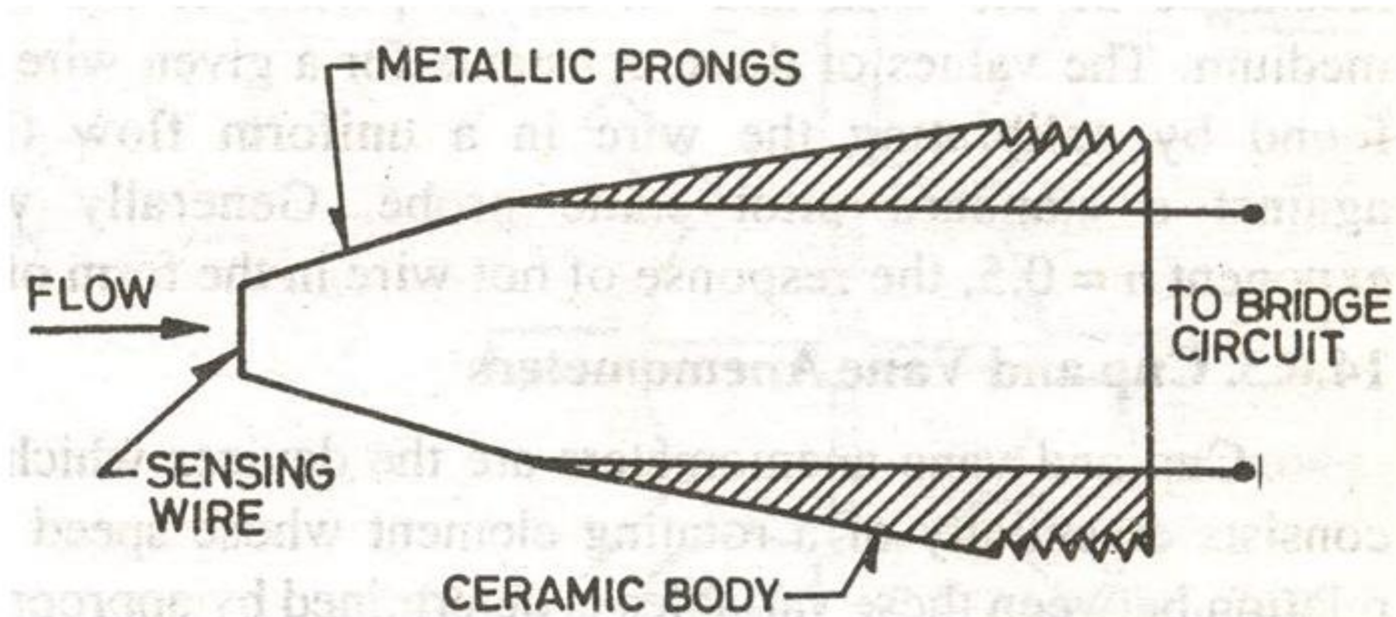
**n** ≈ **1/2** at **high Reynolds numbers**.

- The hot wire normally operates in the **high-Reynolds-number** range but should be calibrated in each situation to find the best-fit a, b, and n.
- The wire can be operated either at **constant current I**, so that resistance R is a measure of V, or at **constant resistance R** (constant temperature), with I a measure of velocity. In either case, the output is a **nonlinear function of V**, and the equipment should contain a linearizer to produce convenient velocity data.
- Because of its **frailty** (infirmity or weakness), the hot wire is **not suited to liquid flows**, whose high density and entrained sediment will knock the wire **right off**. A more stable yet quite sensitive alternative for liquid-flow measurement is the **hot-film anemometer** (Fig. 4.7f).

## Cont…

- A thin metallic film, usually platinum, is plated onto a relatively thick support which can be a wedge, a cone, or a cylinder. The operation is similar to the hot wire. The cone gives best response but is liable to error when the flow is yawed/changed to its axis.
- Hot wire anemometer has been considered as a satisfactory approach to the measurement of **mean** and **fluctuating velocity components** in a flow field.
- The sensor is 5 micron diameter **platinum-tungston wire** welded between the two prongs of the probe and **heated electrically** as a part of wheat stone bridge circuit as shown in fig. 4.10.

Fig. 4.10: Hot wire probe



# Cont…

- When the **probe** is introduced in to the **flowing fluid**, it tends to be **cooled** by the **instantaneous velocity** fluid velocity and consequently there is a tendency for the **electrical resistance** to **diminish**. Since as temperature of the wire decreases the resistance also decreases
- The rate of cooling of the hot wire depends up on
  - ✓ The dimensions and physical properties of the wire
  - ✓ The difference in temperature between the wire and the fluid
  - ✓ The physical properties of the fluid, and
  - ✓ The stream velocity of the fluid under measurement
- **For a simple hot wire anemometer**, the first **three conditions** are effectively **constant** and the **instrument response** is then a **direct measure of the flow velocity**.
- Depending on the associated electronic equipment, a hot wire set may be operated in two ways (shown in fig 4.11)
  - ✓ Constant current mode, and
  - ✓ Constant temperature mode

## Cont…

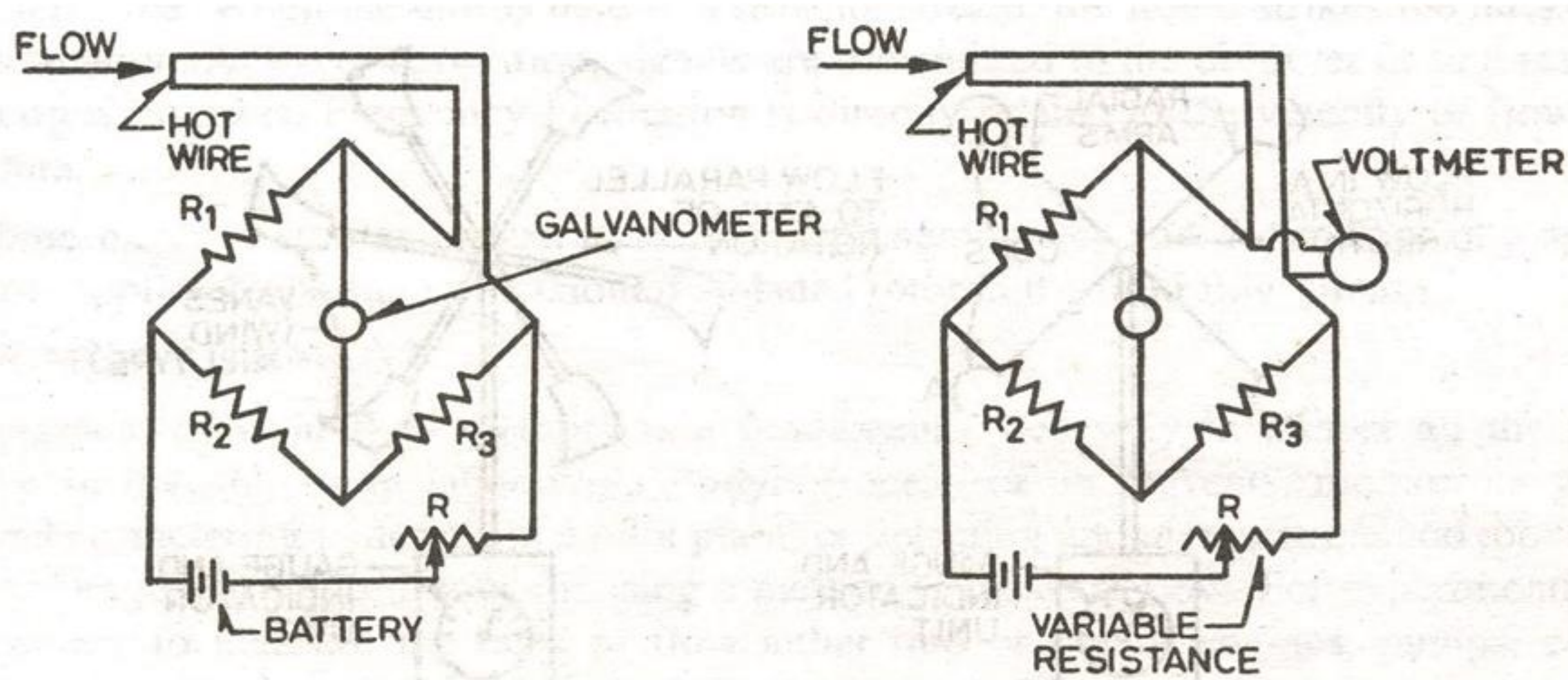



Fig. 4.11: Hot wire anemometer bridge circuits

**Constant current mode**:

- Where the **heating current** (i.e. voltage) across the bridge is **kept constant**.
- Initially the circuit is adjusted such that the **galvanometer reads zero** when the heated wire lies in a **stationary air**. When the **air flows**, the **hot wire cools**, the **resistance changes** and the **galvanometer deflects**. The galvanometer deflection is amplified, measured and correlated with air **velocity** by prior calibration.

**Constant temperature mode**:

- Wherein operating resistance of the wire and hence its temperature is maintained constant.
- In the **event of the tendency of the hot wire** to **cool** by the flowing fluid, an **external bridge voltage is applied** to the wire to maintain sensibly **constant temperature**. The bridge voltage is varied so as to bring the galvanometer needle to zero; the reading on the voltmeter is recorded and correlated with the air velocity.
- The response of the hot wire placed normal to flow stream with velocity V is known to be described by Equation (d) or it may also described as:

$$E^2 = {E_o}^2 + BV^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (e)$$

Where E - is the instantaneous value of the bridge voltage

$E_o$ and B are constants depending on the cold resistance of the wire and on the properties of the fluid medium. The values of these constants for a given wire are found by calibrating the wire in a uniform flow field against a standard pitot static probe.

V – is the fluid velocity to be determined

**3. Cup and vane anemometer**:

- These are devices which are used to measure the **speed of air movements** (fig. 4.7a).
- These device consists of a **rotating element** whose speed of rotation **varies with the velocity of flow**.
- In cup anemometer **cups** are attached to radial arms mounted on a shaft. **Drag forces** are set up on **these cups** when a flow stream in the plane of rotation approaches the unit from **any direction**.
- For the arrangement shown in fig 4.12a, the **drag on the cup A** (cup with its open end facing towards the stream) is **greater than that on cup B** (cup with round face towards the stream). The **resultant torque rotates the assembly** in the anticlockwise direction. The number of revolution is **read from a dial** for a given period of time, and the **frequency of rotation gives** a measure of the **average speed of air** in the region traversed by the air.

# Cont…

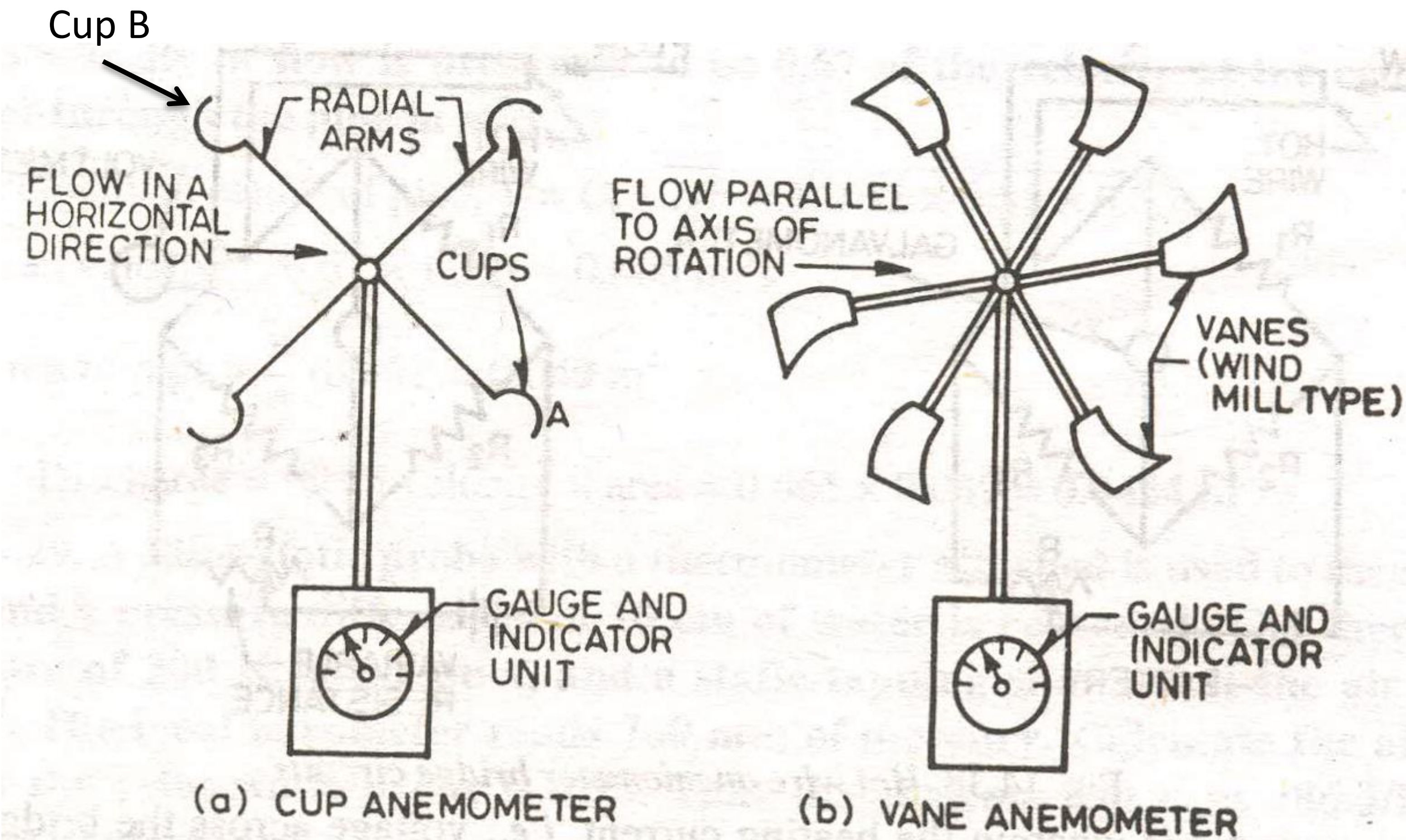



**Fig. 4.12**: cup and vane anemometer

- In a vane anemometer, vanes of the wind mill type are mounted in a support so that the fluid flow is parallel to the axis of rotation. The rotor derives a low friction gear train that in turn drives a pointer that indicates the wind speed on a dial.

# Cont…

- The cup anemometer is usually mounted on a rigid shaft where as a vane type is held in hand while reading are being taken.
- The cup type unit is best for relatively low speeds where as the vane type measures large speeds more accurately.
- Experiments indicate that **provided the wind speed is not too large**, the relation between **wind speed** and **angular velocity of cups/vanes** is **linear**.

**4. Current and turbine meters**:

- **Current** and **turbine meters** are devices which measures the **speed of water movement** (fig. 4.13).
- A current meter consists of a horizontal wheel on which are fixed the conical buckets of V-shaped vanes.
- The unit is suspended in to the flow stream by a suspension cable which is held taut (in tension) by a streamlined weight.
- Horizontal positioning (placement of the unit along the flow direction) is ensured by a streamlined tail vane.

# Cont…

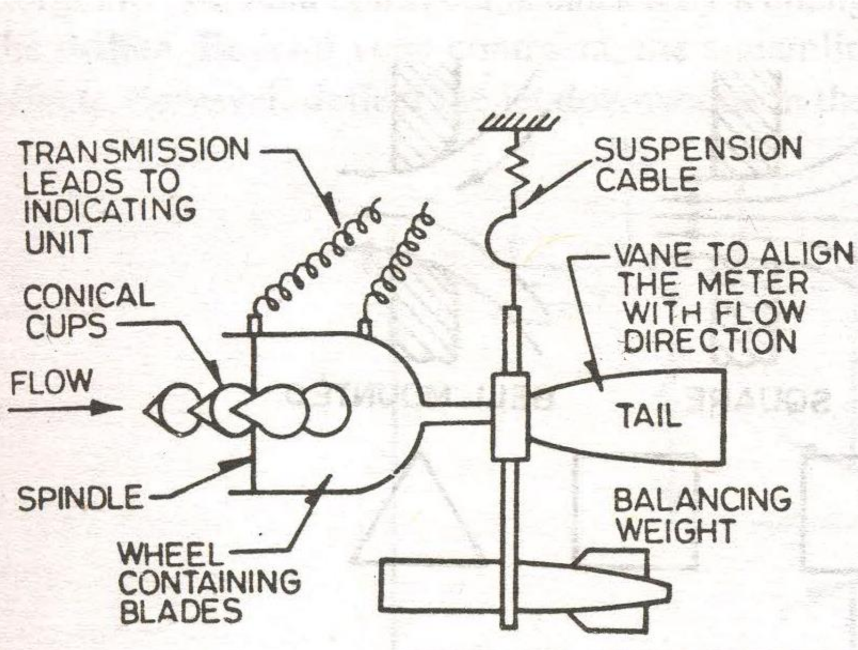



**Fig. 4.13a**: Current meter

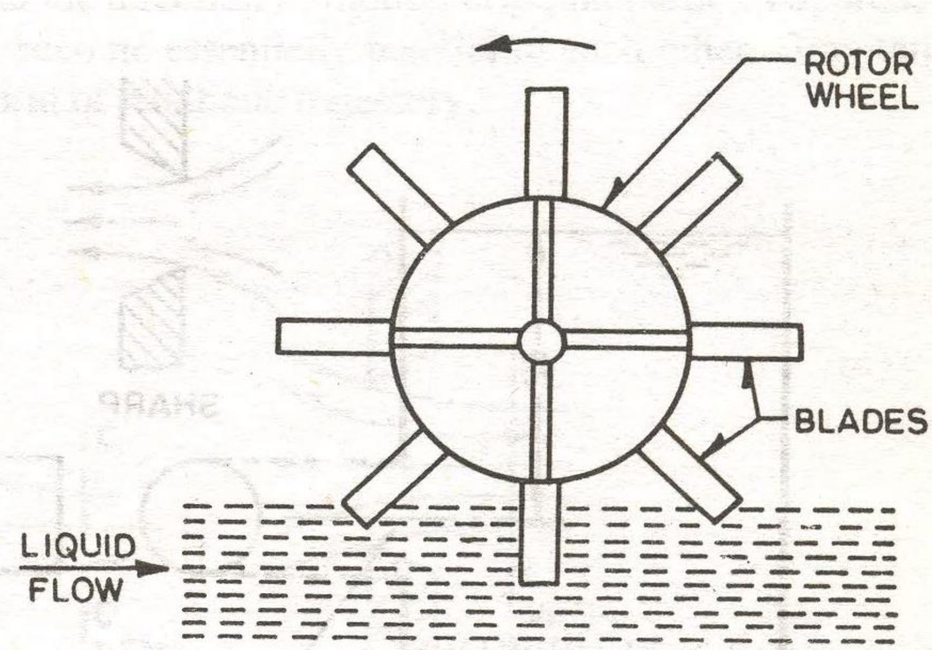



**Fig 4.13b**: Turbine meter

- When the unit is held in a flowing stream, the liquid strikes the buckets and that sets **the wheel** in **rotation**. At every revolution, **signals are transmitted to the observer** or to a **revolution counter through electrical contacts**. Frequency of rotation is directly related to the velocity of the flow by appropriate calibration data.
- A **turbine meter** is **similar in operation to a cup anemometer** and depends on the application of an eccentric force applied by **partial immersion of a blade rotor** in the fluid flow stream.

# Discharge measurement

- A major application area of fluid mechanics is the determination of the **flow rate of fluids**, and numerous devices have been developed over the years for the purpose of **flow metering**.
- Flow meters range widely in their **level of sophistication**, **size**, **cost**, **accuracy**, **versatility**, **capacity**, **pressure drop**, and the **operating principle**.
- We will look at an overview of the different meters commonly used to measure the flow rate of liquids and gases flowing through pipes or ducts. Here, we limit our consideration to incompressible flow.
- Some flow meters measure the flow rate **directly** by discharging and recharging a measuring chamber of known volume continuously and keeping track of the number of discharges per unit time.
- But most flow meters measure the flow rate **indirectly** - they measure the average velocity V and determine the volume flow rate using:

$$\dot{V} = VA_C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(a)$$

*where* $\dot{V}$ *is the volume flow rate,*

$V -$ *is the average velocity and*

$A_c$ *is the cross* $-$ *sectional area of flow*

## Cont…

- Therefore, **measuring the flow rate** is usually done by **measuring flow velocity**, and most flow meters are simply called **velocimeters** used for the purpose of metering flow.
- The velocity in a pipe varies from zero at the wall to a maximum at the center, and it is important to keep this in mind when taking velocity measurements.
  - ✓ For laminar flow, for example, **the average velocity** is half the centerline velocity. But this is not the case in turbulent flow, and it may be necessary to take the weighted average of several local velocity measurements to determine the average velocity.
- The flow rate measurement techniques range from **very crude (rough or simple or basic)** to **very elegant (well-designed)**.
- The flow rate of water through a **garden hose**, for example, can be measured simply by collecting the water in a bucket of known volume and dividing the amount collected by the collection time (Fig. 4.14).

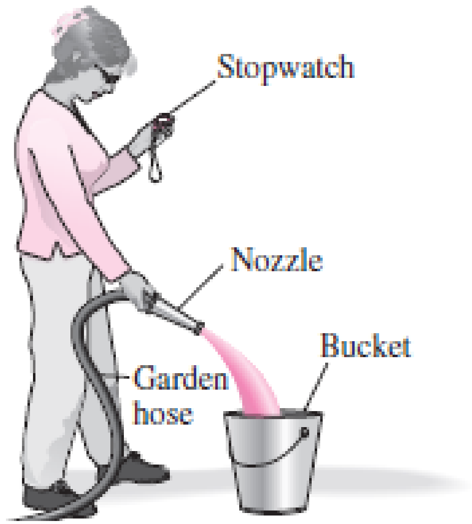



**Fig 4.14:** A primitive (but fairly accurate) way of measuring the flow rate of water through a garden hose involves collecting water in a bucket and recording the collection time.

- Generally beyond the simple flow rate measurement techniques as shown in fig 4.14, flow rates can be measured using:
  - ✓ **Obstruction Flow meters**: Common types of obstruction meters are Orifice meter, Venturi meter, and Flow nozzle or Nozzle Meters
  - ✓ **Variable-Area Flow meters (Rota-meters)**

## 1. Obstruction Flow meters

- Consider an incompressible steady flow of a fluid in a horizontal pipe of diameter D that is **constricted** to a flow area of diameter d, as shown in Fig. 4.15.

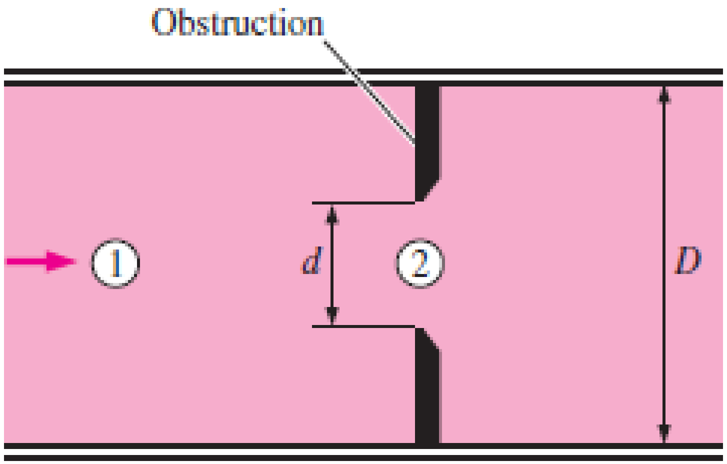



**Fig 4.15:** Flow through a constriction in a pipe.

- The mass balance and the Bernoulli equations between a location before the constriction (point 1) and the location where constriction occurs (point 2) can be written as:

*Mass balance:* $\dot{V} = A_1V_1 = A_2V_2 \quad \rightarrow \quad V_1 = (A_2/A_1)V_2 = (d/D)^2V_2$ ……………… (b)

*Bernoulli equation* ($z_1 = z_2$): $\dfrac{P_1}{\rho g} + \dfrac{V_1^2}{2g} = \dfrac{P_2}{\rho g} + \dfrac{V_2^2}{2g}$ ……………… (c)

Combining Eqs. (b) and (c) and solving for velocity $V_2$ would give

Cont…

*Obstruction (with no loss):* $$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$ ………………………… (d)

where $\beta = d/D$ is the diameter ratio.

Once $V_2$ is known, the flow rate can be determined from

$$\dot{V} = A_2 V_2 = (\pi d^2/4) V_2.$$ ………………………… (e)

- This simple analysis shows that the flow rate through a pipe can be determined by **constricting** the flow and measuring the **decrease in pressure** due to the **increase in velocity** at the **constriction** site. Noting that the pressure drop between two points along the flow can be measured easily by a **differential pressure transducer** or **manometer**, it appears that a simple flow rate measurement device can be built by obstructing the flow. Flow meters based on this principle are called **obstruction flow meters and are widely used to** measure flow rates of gases and liquids.

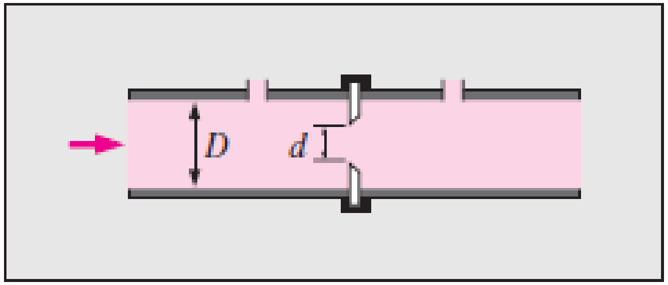
- The velocity in Eq. (d) is obtained by assuming **no loss**, and thus it is the maximum velocity that can occur at the constriction site. **In reality**, some pressure losses due to **frictional effects** are **inevitable**, & thus the velocity will be **less**. Also, the fluid stream will continue to contract **past the obstruction**, and the **vena contracta area** is less than the flow area of the obstruction.
- Both these losses can be accounted for by incorporating a **correction factor** called the **discharge coefficient $C_d$** whose value (which is less than 1) is determined experimentally. Then the flow rate for obstruction flow meters can be expressed as:

*Obstruction flowmeters:* $$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \quad \text{............................ (f)}$$

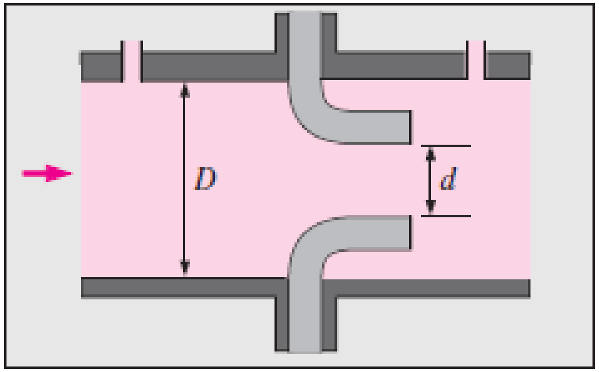
where $A_o = A_2 = \Pi d^2/4$ is the cross-sectional area of the hole or the constriction and $\beta = d/D$ is the ratio of hole diameter to pipe diameter.

- The value of $C_d$ depends on both $\beta$ and the Reynolds number $Re = V_1 D/\nu$, where $\nu$ is the kinematic viscosity.

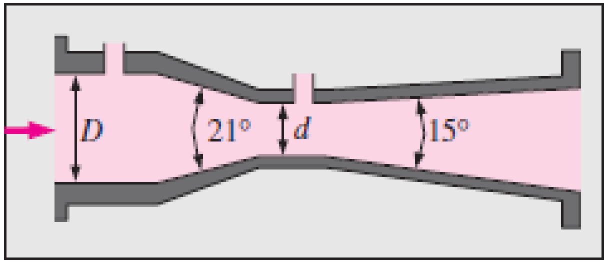
- Of the numerous types of obstruction meters available, those most widely used are **orifice meters**, **flow nozzles**, and **Venturi meters** (Fig. 4.16).




(a) Orifice meter




(b) Flow nozzle




(c) Venturi meter

**Fig 4.16:** Common types of obstruction meters

- The experimentally determined data for **discharge coefficients $C_d$** are expressed as

*Orifice meters:* $$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^{8} + \frac{91.71\beta^{2.5}}{\mathrm{Re}^{0.75}} \quad \text{..............................} \; (g)$$

*Nozzle meters:* $$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\mathrm{Re}^{0.5}} \quad \text{..............................} \; (h)$$

- These relations are valid for $0.25 < \beta < 0.75$ and $10^4 < \mathrm{Re} < 10^7$. The precise values of $C_d$ depend on the particular design of the obstruction, and thus the manufacturer's data should be consulted when available.

- For flows with **high Reynolds numbers** (Re > 30,000), the value of $C_d$ can be taken to be 0.96 for **flow nozzles** and 0.61 for **orifices**.
- Owing to its streamlined design, the **discharge coefficients** of Venturi meters are very high, ranging between 0.95 and 0.99 (the higher values are for the higher Reynolds numbers) for most flows. In the absence of specific data, we can take $C_d = 0.98$ for Venturi meters.
- Example: Measuring Flow Rate with an Orifice Meter

The flow rate of methanol at 20°C ($\rho$ = 788.4 kg/m$^3$ and $\mu$ = 5.857 × $10^{-4}$ kg/m · s) through a 4-cm-diameter pipe is to be measured with a 3-cm-diameter orifice meter equipped with a mercury manometer across the orifice place, as shown in Fig. 8–60. If the differential height of the manometer is read to be 11 cm, determine the flow rate of methanol through the pipe and the average flow velocity.

**SOLUTION** The flow rate of methanol is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate and the average flow velocity are to be determined.

***Assumptions*** **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is $C_d = 0.61$.

***Properties*** The density and dynamic viscosity of methanol are given to be $\rho = 788.4$ kg/m³ and $\mu = 5.857 \times 10^{-4}$ kg/m · s, respectively. We take the density of mercury to be 13,600 kg/m³.

***Analysis*** The diameter ratio and the throat area of the orifice are

$$\beta = \frac{d}{D} = \frac{3}{4} = 0.75$$

$$A_0 = \frac{\pi d^2}{4} = \frac{\pi (0.03\text{ m})^2}{4} = 7.069 \times 10^{-4}\text{ m}^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{Hg} - \rho_{met})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{Hg} - \rho_{met})gh}{\rho_{met}(1-\beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{Hg}/\rho_{met} - 1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (7.069 \times 10^{-4}\text{ m}^2)(0.61)\sqrt{\frac{2(13{,}600/788.4 - 1)(9.81\text{ m/s}^2)(0.11\text{ m})}{1 - 0.75^4}}$$

$$= \mathbf{3.09 \times 10^{-3}\text{ m}^3/\text{s}}$$

which is equivalent to 3.09 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{3.09 \times 10^{-3}\text{ m}^3/\text{s}}{\pi (0.04\text{ m})^2/4} = \mathbf{2.46\text{ m/s}}$$

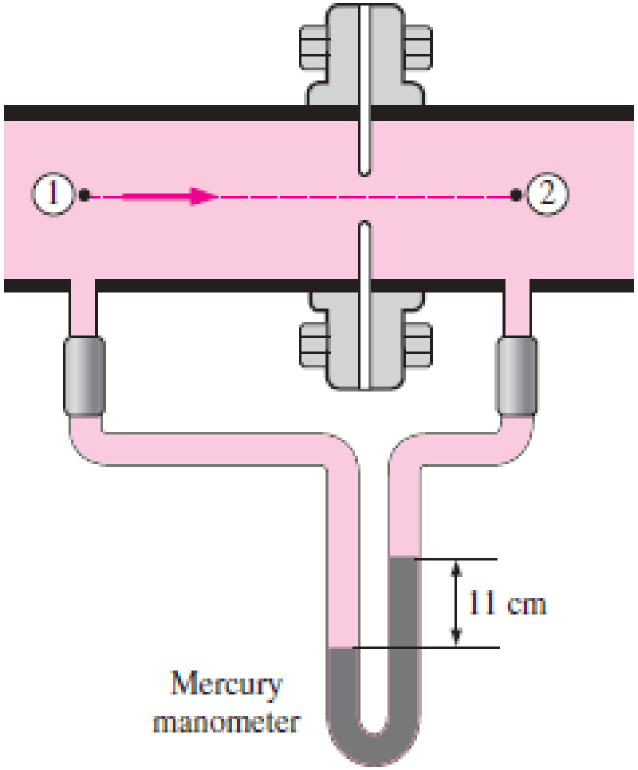



**FIGURE 8–60**
Schematic for the orifice meter considered in Example 8–10.

*Discussion* The Reynolds number of flow through the pipe is

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(788.4\ \mathrm{kg/m^3})(2.46\ \mathrm{m/s})(0.04\ \mathrm{m})}{5.857 \times 10^{-4}\ \mathrm{kg/m \cdot s}} = 1.32 \times 10^5$$

Substituting $\beta = 0.75$ and $\mathrm{Re} = 1.32 \times 10^5$ into the orifice discharge coefficient relation

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\mathrm{Re}^{0.75}}$$

gives $C_d = 0.601$, which is very close to the assumed value of 0.61. Using this refined value of $C_d$, the flow rate becomes 3.04 L/s, which differs from our original result by only 1.6 percent. Therefore, it is convenient to analyze orifice meters using the recommended value of $C_d = 0.61$ for the discharge coefficient, and then to verify the assumed value. If the problem is solved using an equation solver such as EES, then the problem can be formulated using the curve-fit formula for $C_d$ (which depends on the Reynolds number), and all equations can be solved simultaneously by letting the equation solver perform the iterations as necessary.

## 2. Variable-Area Flow meters (Rota-meters):

- A simple, reliable, inexpensive, and easy-to-install flow meter with low pressure drop and no electrical connections that gives a direct reading of **flow rate** for a wide range of liquids and gases is the variable-area flow meter, also called a rotameter or float meter.
- A variable-area flow meter consists of a **vertical tapered conical transparent tube made of glass or plastic** with a **float** inside that is free to move, as shown in Fig. 4.17.
  - ✓ As fluid flows through the tapered tube, the float rises within the tube to a location where the float weight, drag force, and buoyancy force balance each other and the net force acting on the float is zero.
  - ✓ The **flow rate** is determined by simply **matching the position of the float** against the **graduated flow scale** outside the tapered transparent tube.
- We know from experience that high winds knock down trees, break power lines, and blow away hats or umbrellas. This is because the **drag force** increases with flow velocity.
- The weight and the buoyancy force acting on the float are constant, but the **drag force** changes with **flow velocity**.

## Cont…

- Also, the velocity along the tapered tube decreases in the flow direction because of the increase in the cross-sectional area. There is a certain velocity that generates enough drag to balance the float weight and the buoyancy force, and the location at which this velocity occurs around the float is the location where the float settles. The degree of tapering of the tube can be made such that the vertical rise changes linearly with flow rate, and thus the tube can be calibrated linearly for flow rates. The transparent tube also allows the fluid to be seen during flow.
- There are numerous kinds of variable-area flow meters.
  - ✓ The gravity-based flow meter, and
  - ✓ The spring-opposed flow meters


**Fig 4.17:** Two types of variable-area flow meters: (a) an ordinary gravity-based meter and (b) a spring-opposed meter.

## Cont…

- The **gravity-based flow meter** discussed above must be positioned v**ertically**, with fluid entering from the bottom and leaving from the top.
- In **spring-opposed flow meters**, the drag force is balanced by the spring force, and such flow meters can be installed horizontally.

# Viscosity measurement

- **Viscosity** is measured with a device called **Viscosimeters** or **viscometers**.
- The operation of all the viscometers depends upon the existence of laminar flow under certain controlled and reproducible conditions.
- There are different types of viscometers available
  - ✓ **Capillary tube viscometer**
  - ✓ **Efflux viscometers**
  - ✓ **Falling sphere viscometer**
  - ✓ **Rotating cylinder viscometer**

- The capillary tube viscometer as shown in fig 4.18, represents the basic principle of a capillary tube viscosity wherein the viscosity measurements are based on **Poiseuilli's relation**:

$$h_f = \frac{p_1 - p_2}{w} = \frac{128\,\mu\,Q\,l}{\pi\,w\,d^4} = \frac{32\,\mu\,V\,l}{w\,d^2} \quad \text{.............................. (a)}$$

✓ This equation (i.e. equation (a)) is valid only for laminar flow conditions in a circular pipe.

**Where**

Q – is the flow rate of the liquid

W – is the specific weight of the liquid

hf – is the head loss over length *l* of a capillary tube of diameter d.

V – is the fluid velocity

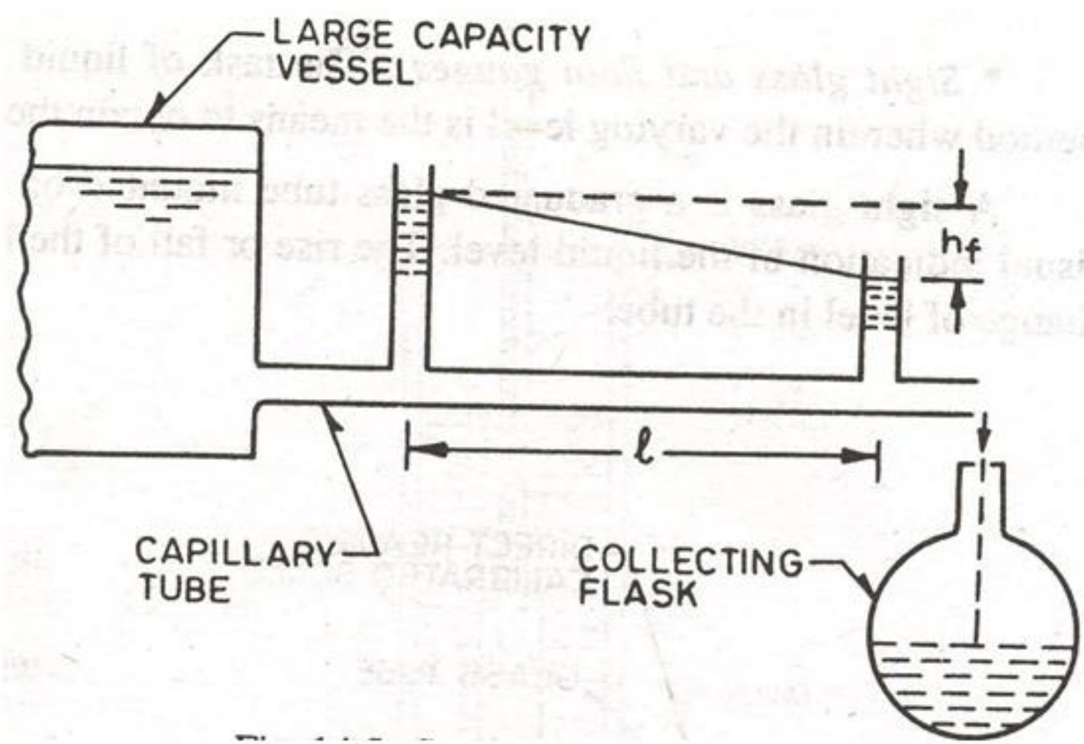



**Fig. 4.18**: Capillary tube viscometer

- Measurement of the flow rate Q and the head loss $h_f$ = dP/w across two cross-section in a capillary would thus give the **liquid viscosity**.

Example 1: . **A fluid of density 1020 kg/m$^3$ was made to pass through a tube, length 35 cm and bore 0.1 cm, under a head of 18 cm. A discharge equivalent to 50 cm$^3$ was collected in a time period of 500 seconds. Make calculations for the dynamic viscosity of the fluid and comment upon the nature of flow through the tube.**

**Solution :** $Q = \dfrac{50 \times 10^{-6}}{500} = 10^{-7}\ \text{m}^3/\text{s}$ ; $V = \dfrac{10^{-7}}{\frac{\pi}{4}(0.001)^2} = 0.127\ \text{m/s}$

For laminar flow through a pipeline the head loss $h_f$ is given by

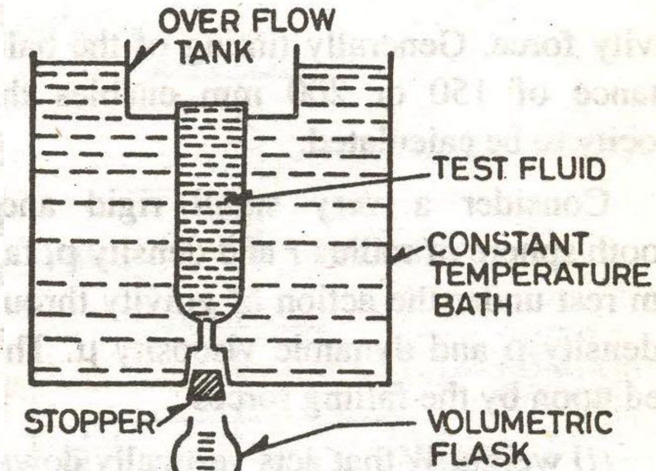
$$h_f = \frac{p_1 - p_2}{w} = \frac{32\,\mu\,V\,l}{w\,d^2} \quad ; \quad 0.18 = \frac{32\,\mu \times 0.127 \times 0.35}{(1020 \times 9.81) \times (0.001)^2}$$

$$\therefore \quad \mu = \frac{0.18 \times (1020 \times 9.81) \times (0.001)^2}{32 \times 0.127 \times 0.35} = 1.266 \times 10^{-3}\ \text{N s/m}^2$$

Flow Reynolds number $R_e = \dfrac{V\,d\,\rho}{\mu} = \dfrac{0.127 \times 0.001 \times 1020}{1.266 \times 10^{-3}} = 102.32$

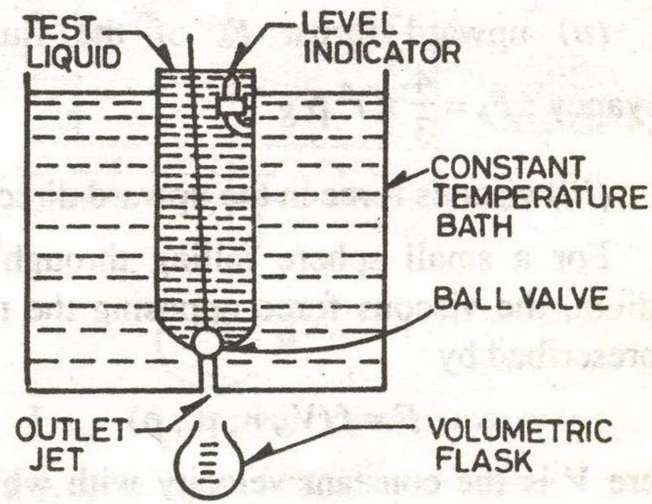
Since $R_e < 2000$, the flow is laminar.

**Efflux viscometers :** These units determine viscosity by noting the time of efflux, under specific conditions, for a fixed volume of fluid through a specific capillary or aperture (standardised orifice or nozzle). As the coefficient of viscosity depends on the temperature of the test fluid, care is taken to ensure that the liquid remains at a uniform constant temperature during the test run. This is accomplished by providing a constant temperature bath around the cylinder which contains the test liquid.



(a) SAYBOLT VISCOMETER

The operation of Saybolt, Redwood and Engler viscometers is based on this principle. In the Saybolt, viscometer, time in seconds for 60 ml of oil to flow through an orifice gives a measure of the oil viscosity which is expressed in term of Saybolt universal seconds. The specified volume for Redwood viscometer is 50 ml and the viscosity is denoted in Redwood seconds. In an Engler viscometer, the time is measured to collect 200 ml of liquid. The results are then expressed in Engler degrees which is the ratio of the time of flow of 200 ml of oil at the test-temperature to the time for the same velocity of water at 20°C.



(b) RED WOOD VISCOMETER

The viscosity in the Engler, Redwood and Saybolt degrees can be converted into kinematic viscosity ν by the empirical relation

$$\nu = At - \frac{B}{t} \quad \text{......................(b)}$$

Where

ν – is the kinematic viscosity of the fluid

A and B – are constants applicable to the viscometer

t – is the time of efflux in seconds.

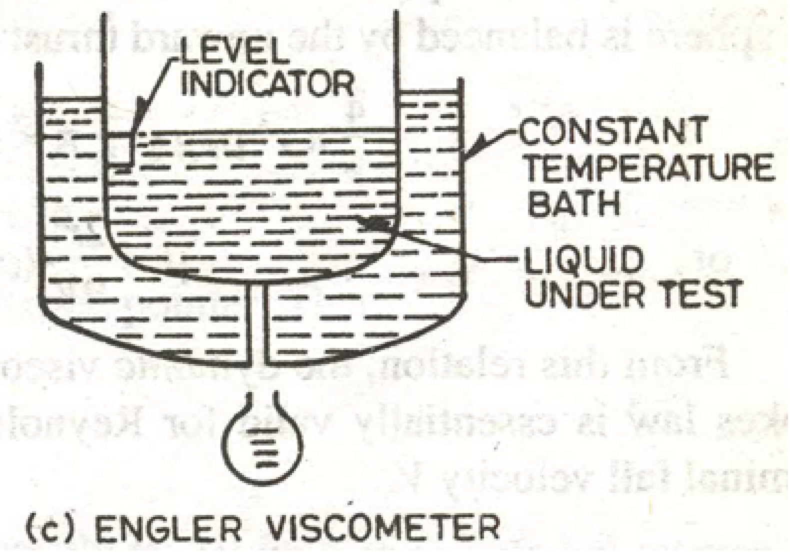



Fig. 4.19: The different Efflux viscometers

- Commonly accepted values of A and B for the different viscometers are given below

| *Viscometer* | *A* | *B* |
|---|---|---|
| Saybolt | 0.24 | 190 |
| Redwood | 0.26 | 172 |
| Engler | 1.47 | 374 |

These laboratory instruments are widely used in the petroleum and allied industries.

**Falling sphere viscometer**:

- A timed fall of ball through liquid in a tube is **proportional** to the absolute viscosity. This forms the operating principles of a falling ball viscometer as shown in fig. 4.20.

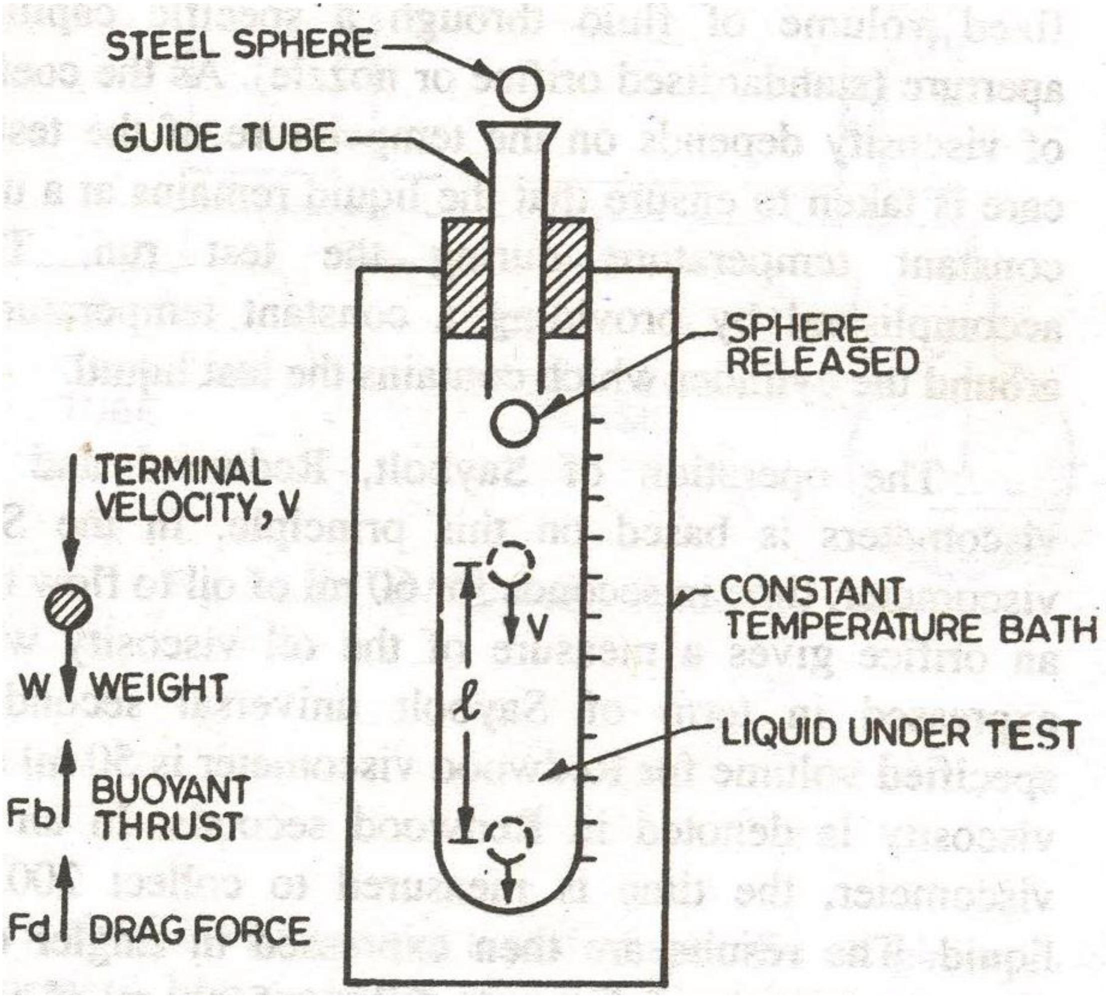



Fig. 4.20: falling sphere viscometer

Cont…

A steel ball or sphere is released into the test liquid and it accelerates under the gravitational force until it reaches a maximum (terminal) velocity *V* when the buoyant and the viscous drag forces balance the gravity force.

- Generally timing of the ball through a distance of 150 or 200mm enables the terminal velocity to be calculated
- Consider a very small rigid and perfectly smooth sphere of radius r and density $\rho_s$ falling freely from rest under the action of gravity g through a liquid of density $\rho$ and dynamic viscosity $\mu$. Here the sphere is acted up on three forces:

✓ **The weight of the sphere**, W: which acts vertically downwards:

$$W = \frac{4}{3}\pi r^3 \rho_s g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

✓ **An upward trust force $F_b$ of the fluid due to buoyancy**, and

$$F_b = \frac{4}{3}\pi r^3 \rho g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

✓ **The viscous force in the upward direction**: for a small sphere falling through a viscous medium, the viscous force opposing the motion can be prescribed by:

$$F = f(V, r, \mu, \rho)$$

*where* $V$ *is the velocity with which the sphere moves through the fluid*

$r$ *– is the radius of the sphere*

$\mu$ *– is the dynamic viscosity of the fluid and*

$\rho$ *– is the fluid density*

- Dimensional analysis would yield the viscous force as:

$$F = 6\pi\mu r V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (e)$$

Equation (e) is known as the **Stokes law**

The sphere acquires the constant velocity, called the terminal velocity, when the gravitational pull on the sphere is balanced by the upward thrust of the liquid and the viscous drag on it. In equilibrium position,

*upward trust force + the drag force − the weight of the sphere = 0*

$$F_b + F - W = 0$$

$$\frac{4}{3}\pi r^3 \rho_s g + 6\pi\mu r V - \frac{4}{3}\pi r^3 \rho g = 0$$

*Re − arranging would give*

$$\mu = \frac{2r^2}{9V}(\rho_s - \rho) g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (f)$$

From this relation, the dynamic viscosity $\mu$ of the fluid can be determined. It may be pointed out that Stokes law is essentially valid for Reynolds number below 0.1 and where the wall has no effect on the terminal fall velocity $V$.

Example 2 **In a falling-sphere viscometer, a lubricating oil of density 800 kg/m$^3$ was placed in a 30 mm inside diameter tube. A 2 mm diameter steel ball of density 7700 kg/m$^3$ was found to travel between the 180 and 40 mm marks in 21.5 seconds. Make calculations for the dynamic and kinematic viscosity of the oil.**

**Solution :** Terminal velocity $V = \dfrac{180 - 40}{21.5} = 6.51 \text{ mm/s} = 0.00651 \text{ m/s}$

Substituting the given data in Stokes law ; $\mu = \dfrac{2}{9}\dfrac{r^2}{V}(\rho_s - \rho)\, g$

$$\mu = \frac{2}{9}\,\frac{(1 \times 10^{-3})^2}{0.00651}\,(7700 - 800) \times 9.81 = 2.31 \text{ N s/m}^2 = 23.1 \text{ poise}$$

Kinematic viscosity $\nu = \dfrac{\mu}{\rho} = \dfrac{2.31}{800} = \mathbf{0.002887\ m^2/s}$